STRESSES AND FAULTING – EXPLORING BRITTLE FAILURE IN THE SHALLOW EARTH

OVERVIEW

This module is designed for students in an introductory structural geology course. While key concepts are described here, it is assumed that the students will have access to a good textbook to augment the information presented here.

Learning goals:

- Understand the role of gravity and rock properties in producing stresses in the shallow Earth.
- Graphically represent stress states using Mohr diagrams.
- Determine failure criteria from the results of laboratory experiments.
- Explore the interaction of gravity-induced and tectonic stresses on fault formation.
- Apply models of fault formation to predict fault behavior in two natural settings: San Onofre Beach in southern California and Canyonland National Park in Utah.

The module is implemented entirely using Microsoft Excel. This program was selected due to its widespread availability and relative ease-of-use. It is assumed that students are familiar with using equations and graphing tools in Excel.

INTRODUCTION

Tectonic forces produce a wide variety of observable geologic phenomenon, including earthquakes and faulting. Approximately 500,000 earthquakes[1] are recorded on a world-wide network of seismic stations each year; ~100,000 of those can be felt, and ~100 cause damage. In southern California alone, ~10,000 earthquakes occur annually. Field- and laboratory-based observations of fault formation and movement provide information used to identify equations that describe the mechanical behavior of faults. Once these relations are known, they can be applied to predict the behavior of faults in response to various stress conditions. In this module, you will explore the application of classical Newtonian mechanics to the shallow layers of the Earth in order to understand the formation and movement of faults.

STATEMENT OF THE PROBLEM

In this module you will develop some simple models to evaluate the state of stress in the shallow Earth, including the relative contributions of gravity-induced and tectonic stresses. You will then use these models to evaluate conditions for fault formation in two areas: southern California and the Canyonlands area of Utah.

You will address four basic questions in this module:

1. *What stresses occur in the earth due to the effects of gravitational loading?*
2. *What conditions are necessary for brittle failure to occur in rocks?*
3. *When a fault forms, what type of faulting will occur and in what orientation?*
4. *What is the result of combining gravity-induced and tectonic stresses, and how will that affect fault formation?*

BACKGROUND INFORMATION2

<u>Forces and Stresses</u>

Newton's first law of motion says that a body at rest will remain at rest, or a body in motion will remain in motion with a constant velocity, unless it experiences an applied external force. Newton's second law of motion indicates that the acceleration of a body is directly proportional to the force acting on it.

$$F=ma \qquad \text{(Equation 1)}$$

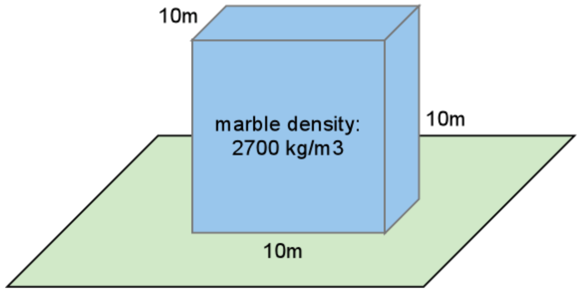
In which F is the force acting on a body, m is the mass of the body, and a is the acceleration of the body. Forces can either be *body* forces (acting throughout the body, such as gravitational and electromagnetic forces) or *surface* forces (acting on a surface of the body, such as across bedding planes or faults). Forces are vectors in that they have both magnitude and direction. Thus forces due to gravitational loading will be in the direction of gravitational pull. The mass of a body can be determined from the density of the material, ρ, and the volume of the body, V:

$$m=\rho V \qquad \text{(Equation 2)}$$

[1] http://earthquake.usgs.gov

[2] For a complete description of forces, stresses, and Mohr diagrams, read the appropriate sections of a Structural Geology textbook; some examples are listed in the References section.

Example 1: Determine the force exerted on the ground by a block of marble whose dimensions measure 10 m on a side.



Solution: A block 10 m on a side has a volume of 1000 m^3. Marble has a density of 2700 kg/m^3. Thus the mass of the block is 27 kg. The force exerted by the block is due to gravitational forces, thus the force at the bottom of the block is equal to the mass of the block times the acceleration of gravity: F = 27 kg * 9.8m/s^2 = 265 kg m/s^2 or 265 Newtons (265 N).

Since the marble block is sitting stationary on the ground, the force of the block on the ground must be balanced by an equal and opposite force from the ground acting on the block. Thus we see that for stationary objects, the forces balance.

When a force acts upon a surface, it produces a stress, σ. This stress is directly proportional to the force and inversely proportional to the surface area. Thus:

$$\sigma = F/A \qquad \text{(Equation 3)}$$

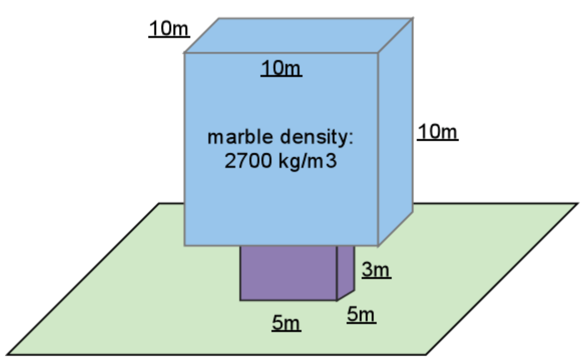
In which *A* is the surface area. Like force, stress is a quantity with both magnitude and direction. In the example above, the stress acting on the surface of the ground beneath the marble block is 2.65 N/m^2 or 2.65 Pascals (2.65 Pa) acting in a vertical direction. Stresses deep in the earth are several orders of magnitude greater than this, so units of *megapascals* (1 MPa = 10^6 Pa) or *gigapascals* (1 Gpa = 10^9 Pa) are commonly used.

Combining Equations 1 through 3 yields

$$\sigma_{vert} = \rho g h \qquad \text{(Equation 4)}$$

where *h* is the height of the block (or depth into the earth) and *g* is the acceleration of gravity. Thus the vertical stress at any depth in the earth can be easily determined if the density and thickness of the overlying material are known.

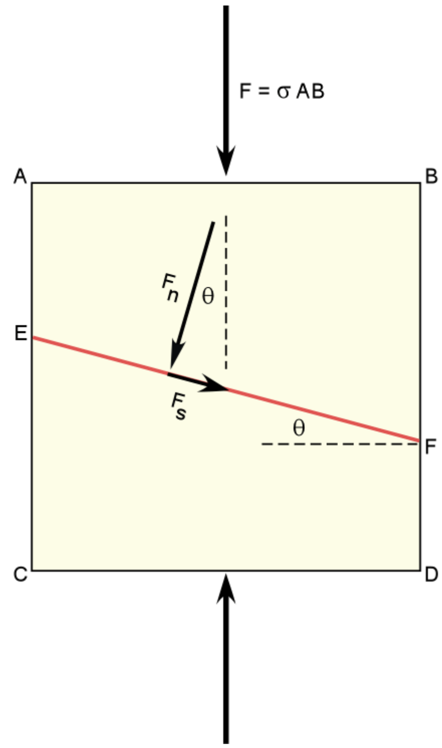
Example 2: Determine the stress at the bottom of the marble block if it were resting on a 3 m high block pedestal with a square base whose sides measure 5 m.



Solution: The force of the marble block is no longer distributed over the entire base of the marble, rather it is supported entirely by the smaller pedestal. The stress acting on the surface in contact between the marble block and the pedestal is thus 10.6 Pa (265 N/25 m^2).

Normal and Shear Stresses

In the examples above, we determined the vertical stress acting on a horizontal surface. What are the stresses on a dipping surface? The force acting in a vertical direction can be broken down into two components: the *normal* force (F_n) acting perpendicular to the tilted surface and the *shear* force (F_s) acting parallel to the tilted surface.



Simple geometric relations[3] indicate that

$$F_n = F\cos\theta = \sigma\, AB\cos\theta \quad \text{(Equation 5a)}$$

$$F_s = F\sin\theta = \sigma\, AB\sin\theta. \quad \text{(Equation 5b)}$$

These forces can be used to determine the *normal stress* and *shear stress* acting on the tilted plane. Since AB = EF cosθ,

$$\sigma_n = F_n/EF = \sigma\cos^2\theta \quad \text{(Equation 6a)}$$

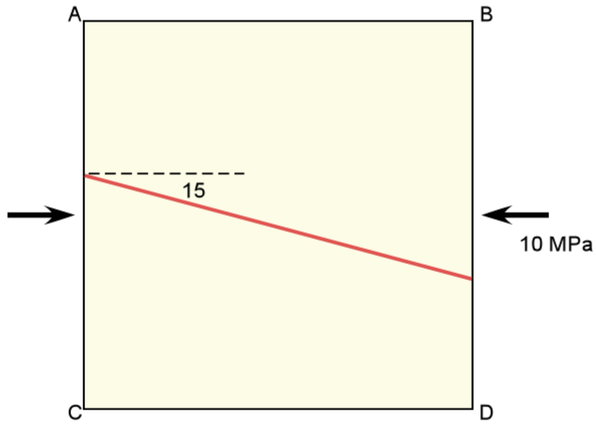
$$\sigma_s = F_s/EF = \sigma\, ½\sin 2\theta. \quad \text{(Equation 6b)}$$

Example 3: What are the normal and shear stresses acting on a plane dipping 15 degrees from horizontal with a vertical stress of 30 MPa?

Solution: θ, the angle between the plane on which σ acts and the dipping plane, is 15º. Thus from Equations 6, σ_n and σ_s are 28 and 7.5 MPa respectively.

[3] From a derivation found in *Earth Structure* by B. A. van der Pluijm and S. Marshak.

Example 4: What are the normal and shear stresses acting on a plane dipping 15 degrees from horizontal with a *horizontal* stress of 10 MPa?

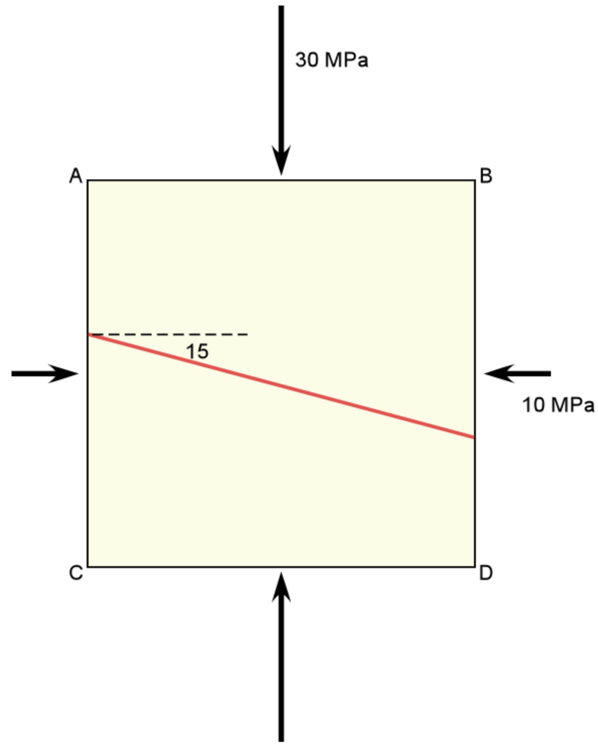


Solution: θ, the angle between the plane on which σ acts (plane AC) and the dipping plane, is 75º. Thus from Equations 6, σ_n and σ_s are 0. 7 and 2.5 MPa respectively.

By convention, geologists represent compression by positive normal stresses and tension by negative normal stresses. Negative shear stresses indicate a *right-lateral* sense of shear (see diagram below) and positive shear stresses indicate a *left-lateral* sense of shear.

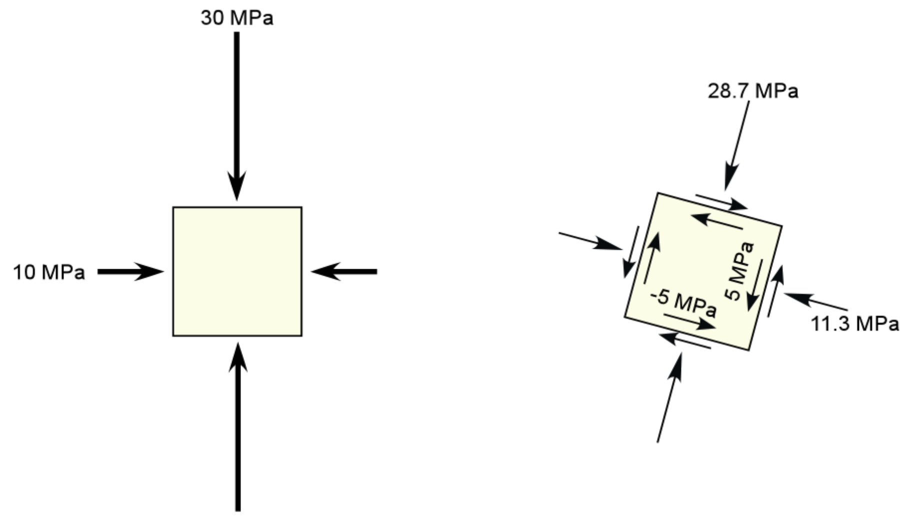
Thus the normal and shear stresses in Example 3 are 30 and -7.5 MPa, and in Example 4 are 0.7 and 2.5 respectively.

Example 5: What are the normal and shear stresses acting on a plane dipping 15 degrees from horizontal with a vertical load of 30 MPa and a horizontal stress of 10 MPa? (shown in figure below)

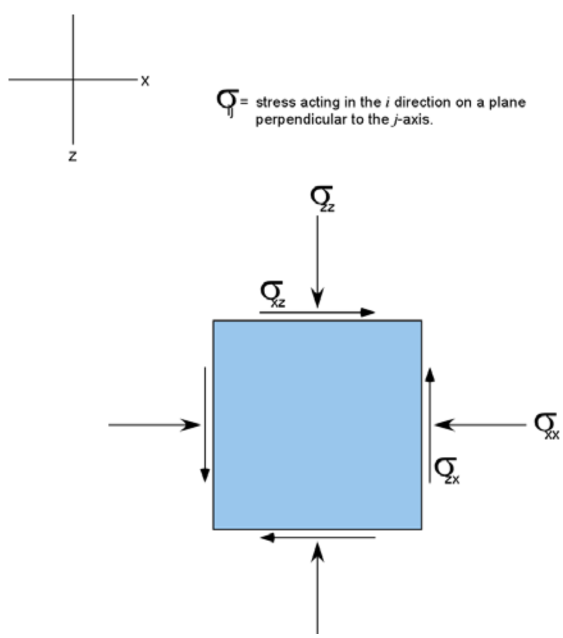


Solution: The normal stress on the dipping plane is 28.7 MPa (28 from the vertical load, 0.7 from the horizontal load) Shear stress on the dipping plane is -5 MPa (-7.5 from the vertical load, 2.5 from the horizontal load).

Any point at depth in the earth will be subjected to both vertical and horizontal loads. Thus we should think about *stress fields*, rather than just unidirectional stresses. Any 2-dimensional stress field can be fully represented by the normal and shear stresses on any two perpendicular planes, and any 3-dimensional stress field can be fully represented by the normal and shear stresses acting on 3 orthogonal (mutually perpendicular) planes. For us to completely describe the 2-D stress field in Example 5, we would need to determine σ_n and σ_s on a plane perpendicular to the one dipping 15º. The figure below shows the same stress field described by two separate sets of perpendicular planes. In the right-hand figure, note that the shear stresses on the two planes are balanced – they are of equal magnitude and opposite sign.



Stresses are readily expressed in reference to a Cartesian coordinate system. Subscripts are used to indicate both the surface on which the stress is acting as well as the direction of the stress. The stresses acting on the plane perpendicular to the z-direction are σ_{zz} and σ_{xz}. σ_{zz} is the stress acting in the z-direction on the surface perpendicular to the z-direction, thus σ_{zz} is the normal stress acting in the z-direction; σ_{xz} is the shear stress acting on the same surface.



Homework Problems 1, 2 and 3.

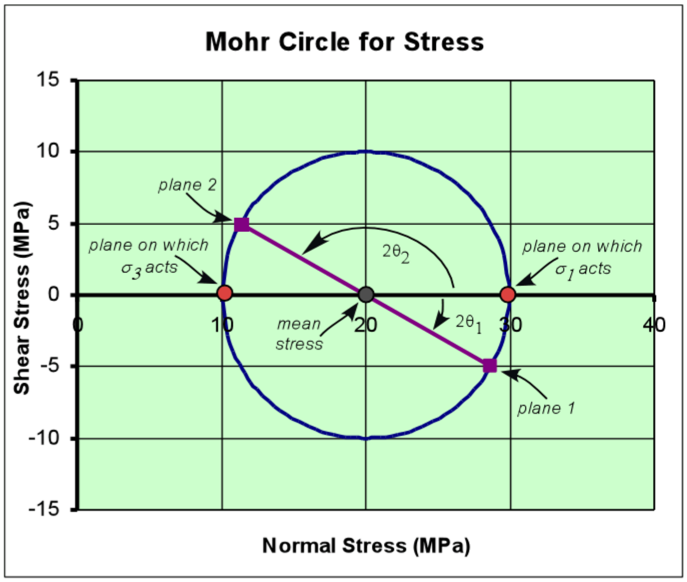
You are now ready to do homework problems 1, 2 and 3. These are located near the end of this module. The first problem explores how σ_{zz} changes with depth and rock type, and introduces you to solving problems using Excel. The second problem explores how elastic properties of rock affect the horizontal stresses introduced by gravitational loading.

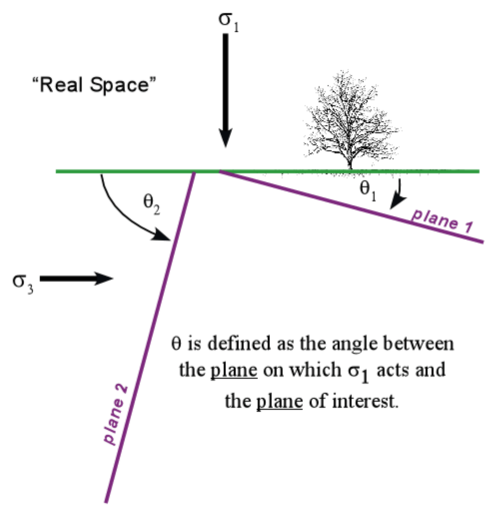
Principal Stresses

For any stress field, there exists one set of orthogonal planes on for which the magnitude of the shear stress is 0. These are referred to as *principal planes*; the normal stresses acting on these planes are the *principal stresses*. By convention, the maximum principal stress is σ_1, the intermediate is σ_2, and the minimum principal stress is σ_3. Thus, $\sigma_1 \geq \sigma_2 \geq \sigma_3$. In the example above, the horizontal and vertical planes are the principal planes, and the vertical and horizontal stresses are the principal stresses.

Mohr Diagram[4] for Stress

Stress states can be graphically represented on a Mohr diagram for stress. On a Mohr diagram, σ_n is plotted on the abscissa and σ_s on the ordinate axis. Points on the circle represent the normal and shear stresses on planes in "real space". The figure below shows how the two planes in Example 5 plot on a Mohr diagram and their positions in the physical world.





Using the Mohr diagram, one can determine equations for σ_n and σ_s in terms of σ_1, σ_3 and θ.

$$\sigma_n = \tfrac{1}{2}(\sigma_1 + \sigma_3) + \tfrac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta \qquad \text{(Equation 7a)}$$

$$\sigma_s = \tfrac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta. \qquad \text{(Equation 7b)}$$

The center of the Mohr diagram is the *mean stress*, the hydrostatic component of the principal stresses. This component of the stress state tends to produce *dilation*, or changes in volume. The radius of the circle is the *deviatoric stress*; this tends to produce non-uniform *distortions* of the rock mass. The diameter of the circle is the *differential stress*, the difference between the principal stress magnitudes.

Homework problems 4 and 5.

You should now be able to do homework problems 4 and 5. In problem 4 you will work with the various components of the Mohr diagram. In problem 5, you will use Mohr diagrams to plot rock strength data for rocks collected from the Hayward Fault in northern California.

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[4] Named after the German engineer Otto Mohr (1835-1918) who first introduced this graphical method in 1882.

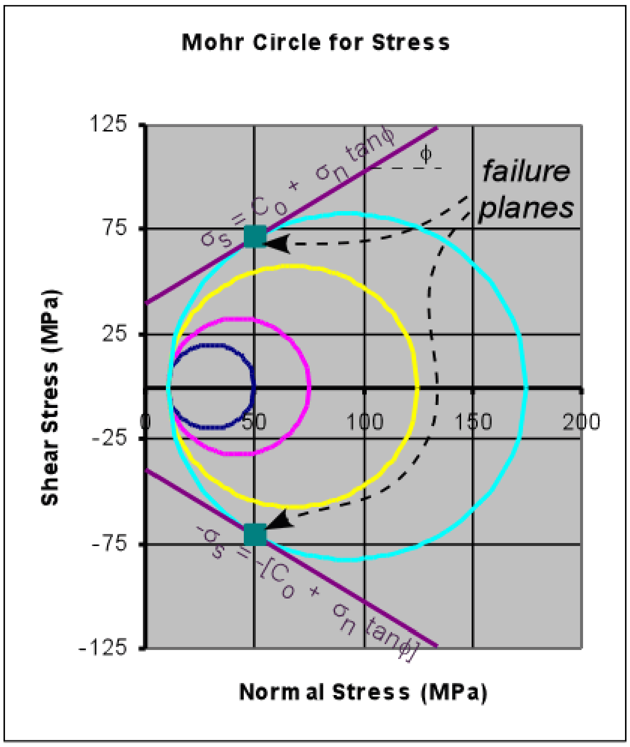
<u>Coulomb Failure and Byerlee's Law</u>

Multiple experiments of rocks failing by brittle faulting in compression have led to the development of the *Coulomb law of failure*. This linear empirical relationship describes the shear stress on a failure plane as a function of normal stress at the moment of initiation of failure:

$$\sigma_s = C_o + \sigma_n \tan\phi \qquad \text{(Equation 8)}$$

in which C_o is the cohesive strength of the rock and $\tan\phi$ is the coefficient of internal friction, a label which has no physical meaning when breaking intact rock; it is sometimes referred to as the Coulomb coefficient and should be thought of as the proportionality constant between normal and shear stresses.

During triaxial rock strength experiments, the rocks remained intact with increasing axial load until the failure criterion is reached (see figure below). The point where the Mohr circle touches the failure envelope first marks the plane on which failure occurs.



Coulomb failure describes breaking intact rock to form new faults. The shear stress required to initiate sliding on pre-existing surfaces (e.g. faults, bedding planes, and joints) is also dependent upon the normal stress acting on the surface. In the 1970's J. Byerlee showed that the shear stress required for slip on pre-existing surfaces is nearly independent of rock type. The empirical relation that best fit the experimental data is referred to as *Byerlee's Law:*

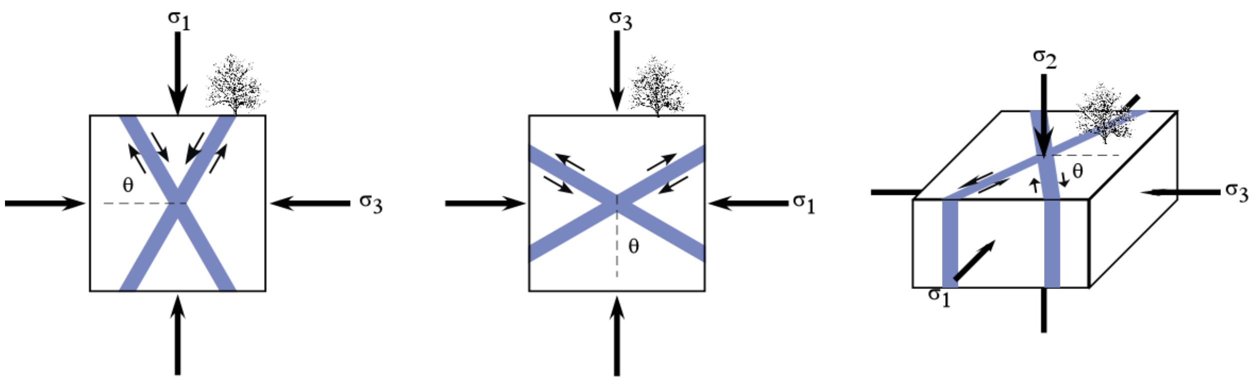
$$\sigma_s = 0.85\,\sigma_n \quad (\text{for } \sigma_n < 200 \text{ MPa}) \qquad \text{(Equation 9a)}$$

$$\sigma_s = 50\text{ MPa} + 0.6\,\sigma_n \quad (\text{for } \sigma_n > 200 \text{ MPa}) \qquad \text{(Equation 9b)}$$

and the *coefficient of friction* (the ratio between shear and normal stresses) for most rocks is 0.6 to 0.85.

Anderson's Theory of Faulting

If new faults form as shear fractures following Coulomb failure, then they generally form at angles ~30º from the σ_1 direction and the intersection of the two potential faults planes is the σ_2 direction. This relationship is referred to as *Anderson's theory of faulting* after the British geologist who first proposed it. The Earth's surface (the contact between ground and air) is a "free surface" in that it cannot transmit a shear stress. Thus the Earth's surface is a principal plane (i.e. the shear stress on this surface is 0), and the vertical stress at the Earth's surface will be one of the principal stresses (σ_1, σ_2, or σ_3). Anderson's theory of faulting states that shallow faults will form in one of three orientations, depending on which principal stress is vertical. If σ_1 is vertical, normal faults should form with dips ~60º; if σ_2 is vertical, strike-slip faults should form with dips ~90º; if σ_3 is vertical, thrust faults should form with dips ~30º. Note that in all three examples, θ is ~60º.



Adding Stress Fields

In problem 2 you determined the effect of changing rock properties on the horizontal and vertical stresses due to gravitational loading. Other stress fields[5] can be added to these as long as the reference axes (x and z) are the same for both fields.

$$\sigma_{ij} = \sigma_{ij1} + \sigma_{ij2} \qquad \text{(Equation 10)}$$

where σ_{ij} represents the stress acting in the i^{th} direction on the plane perpendicular to the j axis; the subscripts 1 and 2 refer to different stress fields.

Homework problems 6 and 7.

You should now be able to do homework problem 6 in which you will determine the failure criteria for the Hayward fault rocks. In problem 7, you will explore what happens to the stresses generated by gravitational loading when a horizontal tectonic stress is present – what is the effect of extending or compressing the lithosphere?

Modeling Tectonic Stresses

In problem 7 you explored depth variations of σ_{zz} and σ_{xx} from the addition a constant horizontal tectonic load. Tectonic loads are not necessarily constant with depth, and they can vary over horizontal distances as well. While many functions can be used to model tectonic stresses, a

[5] Additional stress fields might have tectonic, thermal, or magnetic origins, for example.

simple linear model of tectonic stresses can produce interesting results. For this example, let S_{ij} represent the components of stress due to tectonic loading

$$S_{zz} = a + bx + cz \quad \text{(Equation 11a)}$$

$$S_{xx} = d + ex + fz \quad \text{(Equation 11b)}$$

$$S_{xz} = -(bz + fx) \quad \text{(Equation 11c)}$$

x and z are the reference coordinates and a-f are model parameters.

Homework problem 8.

So far, you have explored changes in stresses with depth only. In problem 8 you will expand your model to 2-dimensions to explore variations in stresses both with depth and with horizontal distance.

Homework problem 9: Project and Written Report.

Having investigated the effects of gravitational and tectonic loading in a two-dimensional model in the previous problem, in this final project you will develop models to investigate the factors controlling the initiation of faulting in the Canyonlands area of Utah.

ACKNOWLEDGMENTS

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SOME USEFUL REFERENCES

Jaeger, J.C. and N.G.W. Cook, *The Fundamentals of Rock Mechanics (3rd edition)*, 593 p., Chapman and Hall, Ltd, London, 1979.

Means, W.D., *Stress and Strain: Basic concepts of continuum mechanics for geologists*, 339 p., Springer-Verlag, New York, 1976.

Scholz, C.H., *The Mechanics of Earthquakes and Faulting (2nd edition)*, 471 p., Cambridge University Press, Cambridge, 2002.

Turcotte, D.L., and G. Schubert, *Geodynamics (2nd edition)*, 450 p., Cambridge University Press, Cambridge, 2002.

<u>San Onofre and the Canyonlands Areas:</u>

McGill, G.E., and A.W. Stromquist, The grabens of Canyonlands National Park, Utah – geometry, mechanics and kinematics. Journal of Geophysical Research, 84, 4547-4563, 1979.

Schlemon, R.J., The Cristianitos fault and Quaternary geology, San Onofre State Beach, California. Geological Society of America Centennial Field Guide – Cordilleran Section, 171-174, 1987.

Schultz-Ela, D.D., and P. Walsh, Modeling of grabens extending above evaporites in Canyonlands National Park, Utah. Journal of Structural Geology, 24, 247-275, 2002.

Sharp, R.P. and A.F. Glazner, *Geology Underfoot in Southern California*, 224 p., Mountain Press, Missoula, 1993.

<u>Some General Structural Geology textbooks</u>

Davis, G.H., and S.J. Reynolds, *Structural Geology of Rocks and Regions (2nd edition)*, 776 p., John Wiley & Sons, Inc., New York, 1996.

Suppe, J., *Principles of Structural Geology*, 537 p., Prentice-Hall, New Jersey, 1985.

Twiss, R.J., and E.M. Moores, *Structural Geology*, 532 p., W.H. Freeman & Company, New York, 1992.

van der Pluijm, B.A., and S. Marshak, *Earth Structure (2nd edition)*, 656 p., W.W. Norton & Company, New York, 2004.